

Generalized holographic dark energy and the IR cutoff problem

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Abstract

We consider a holographic dark energy model, in which both the cosmological-constant (CC) energy density ρ_Λ and the Newton constant G_N are varying quantities, to study the problem of setting an effective field-theory IR cutoff. Assuming that ordinary matter scales canonically, we show that the continuity equation univocally fixes the IR cutoff, provided a law of variation for either ρ_Λ or G_N is known. Previous considerations on holographic dark energy disfavor the Hubble parameter as a candidate for the IR cutoff (for spatially flat universes), since in this case the ratio of dark energy to dark matter is not allowed to vary, thus hindering a deceleration era of the universe for the redshifts $z \gtrsim 0.5$. On the other hand, the future event horizon as a choice for the IR cutoff is being favored in the literature, although the ‘coincidence problem’ usually cannot be addressed in that case. We extend considerations to spatially curved universes, and show that with the Hubble parameter as a choice for the IR cutoff one always obtains a universe that never accelerates or a universe that accelerates all the time, thus making the transition from deceleration to acceleration impossible. Next, we apply the IR cutoff consistency procedure to a renormalization-group (RG) running CC model, in which the low-energy variation of the CC is due to quantum effects of particle fields having masses near the Planck scale. We show that bringing such a model (having the most general cosmology for running CC universes) in full accordance with holography amounts to having such an IR cutoff which scales as a square root of the Hubble parameter. We find that such a setup, in which the only undetermined input represents the true ground state of the vacuum, can give early deceleration as well as late time acceleration. The possibility of further improvement of the model is also briefly indicated.

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A few years ago, Cohen *at al.* [1], motivated by the Bekenstein bound [2] on the maximal possible entropy, showed that systems which did not contain black holes might still allow a self-description by means of conventional quantum field theory, if a certain relationship between the UV and IR cutoffs was obeyed. Applying this limit [1] when the volume under consideration is the universe itself, results in one of the most elegant solutions to the (‘old’) cosmological constant (CC) problem [3]. The limit on the zero-point energy density ρ_Λ in [1] represents a more stringent version of the holographic principle [4, 5]. In short, such a principle states that in the presence of quantum gravity, all of the information contained in a certain volume of space can be represented by a theory that counts degrees of freedom only on the boundary of that region.

Recently, starting from [6], a strong impetus for a more thorough investigation of the Cohen *at al.* bound [1] and its consequences in cosmology was triggered by a widespread consensus about the presently accelerated expansion of the universe [7]. In addition, since the size of the region (providing an IR cutoff) is varying in an expanding universe, ρ_Λ (usually dubbed the holographic energy density) is promoted to a dynamical quantity. Therefore one hopes this might also shed some light on a puzzling ‘coincidence problem’ [8]. One can show that irrespective of the hierarchy between the UV cutoff and particle masses, ρ_Λ generated by vacuum fluctuations is always dominated by UV modes, and that the limit from [1] can be rewritten as

$$\rho_\Lambda(\mu) = \kappa \mu^2 G_N^{-1}(\mu) , \quad (1)$$

where μ denotes the IR cutoff, and the bound from [1] is saturated for $\kappa \simeq 1$. The case when G_N from (1) is also promoted to a dynamical quantity [as explicated in (1)] was considered for the first time in [9], and afterwards in [10]. Such a setup we call a generalized holographic dark energy. By the simplest choice for μ in the form of the present Hubble parameter, one indeed obtains the dark energy density very close to the observed one. Thus, for this choice of the IR cutoff, the parameter κ in (1) should be close to unity. One thus sees how efficiently holography stabilizes the vacuum energy and ameliorates the ‘old’ CC problem by eliminating the need for fine-tuning.

Most authors modeled dark energy from (1) to evolve independently of matter density ρ_m , i.e. to behave as a perfect fluid. In this case, both components evolve according to the w -parameter from their equations of state (EOS). Another modeling of ρ_Λ from (1) is through interacting fields [11], where an interaction between dark energy and matter

fields is postulated; in this setup the components no longer evolve according to their w -parameters. Scaling of ρ_Λ due to a different kind of interaction involving nonstatic G_N was also investigated [9, 10, 12]. It is easy to see why the identification of the IR cutoff with the Hubble distance for spatially flat universes leads to unsatisfactory cosmologies for all the cases above (although the interpretation for each case is different). Indeed, by plugging (1) in the Friedmann equation for flat space one obtains

$$H^2 = \frac{8\pi\kappa}{3}\mu^2(1+r) , \quad (2)$$

where $r = \rho_m/\rho_\Lambda$. Thus, the choice $\mu \sim H$ would require the ratio r to be a constant. This holds irrespective of whether a fluid is perfect or not and irrespective of whether G_N is varying or not. The interpretation of various cases is, however, different. For perfect fluids, $r = \text{const.}$ means that the equation of state for dark energy unavoidably matches that of pressureless matter, $w = 0$. Thus, we cannot explain the accelerating expansion of the present universe. For interacting fluids, one usually can generate accelerated expansion with $r = \text{const.}$ as well as ameliorate the ‘coincidence problem’, but fails to explain that the acceleration era set in just recently and was preceded by a deceleration era at $z \gtrsim 0.5$ (see, however, [11]).

Another studied choice for the IR cutoff was the particle horizon distance [13]. However, this option fails for perfect fluids because it is impossible to obtain a w -parameter characterizing accelerated universes. The most popular choice is by far the future event horizon. This choice seems to work well both for perfect [13] and interacting fluids [14]. Within this choice, however, most approaches suffer from the ‘coincidence problem’. For related works, see [15].

In the present paper we adopt the framework consistent with the generalized holographic energy density. This simplest possibility is to let the matter energy density scale canonically $\sim a^{-3}$, i.e. behaving as a perfect fluid. The scaling of ρ_Λ is then induced by the scaling of G_N (and *vice versa*) and is described by the equation of continuity of the form

$$\dot{G}_N(\rho_\Lambda + \rho_m) + G_N\dot{\rho}_\Lambda = 0 , \quad (3)$$

where overdots denote time derivatives. Eq. (3) means that $G_N T_{total}^{\mu\nu}$ and $T_{matter}^{\mu\nu}$ are separately conserved [16]. The difference of the framework phrased by (3) with respect to other models employing holographic energy density is twofold. Firstly, the scaling of ρ_Λ is not

caused by the energy transfer with the matter component, but goes at the expense of the time-dependent gravitational coupling [18]. Secondly, ρ_Λ in (1)-(3) represents the variable (or interacting) but ‘true’ CC, with the EOS $w_\Lambda \equiv p_\Lambda/\rho_\Lambda$ being precisely -1. Although holographic dark energy is usually modeled in the literature through scalar fields (being in the form of both perfect and interacting fluids), we note that the bound for ρ_Λ from [1] was derived by considering vacuum fluctuations (zero point energies). Since these represent the ‘true’ CC, there is no need for invoking a light scalar field with its potential. Note also that even the static ρ_Λ can fit within the bound (1) if G_N is varying.

The aim of the present paper is threefold. First, we obtain a relation which univocally fixes the IR cutoff, provided our framework is phrased by Eqs. (3) and (1). Thus, if, e.g. the law for $G_N(\mu)$ is specified [or for ρ_Λ , which is the same as both quantities are connected through (1)], then the IR cutoff μ is no longer liable to a free choice, but is fixed. This is in clear contradistinction with the case when the energy transfer is between ρ_Λ and ρ_m . In this case, the equation of continuity is of the form

$$\dot{\rho}_\Lambda + \dot{\rho}_m + 3H\rho_m = 0 . \quad (4)$$

We see from (4) that any deviation from the canonical scaling $\rho_m \sim a^{-3}$ depends decisively on the choice of μ . Hence, with different choices for μ one modulates the interaction between ρ_Λ and ρ_m as described by (4). Our second step is to set the IR cutoff in the scale-fixing relation at $\mu = H$ without *a priori* specifying a law for ρ_Λ , in order to investigate if this choice still remains a bad one for curved universes. Finally, we study compatibility of generic variable CC models with the holographic prediction (1). It is known that such models lead to successful cosmologies when the transfer of energy is of the type as given by (4). However, merging such models with the prediction of gravitational holography leads them to be inconsistent with cosmological observation. On the example of the most general running CC model, we show that the situation can substantially be improved if instead the framework described by (3) is used. We then study observational consequences of such a model.

The scale-fixing relation emerges after inserting the holographic prediction (1) in the equation of continuity (3):

$$\mu = -\frac{G'_N(\mu)\rho_m}{2\kappa} , \quad (5)$$

where the prime denotes differentiation with respect to μ . (Note that (5) cannot be applied

to the case $\rho_m = 0$. In this case, (1) and (3) imply $\dot{\mu} = 0$, so a differentiation with respect to t cannot be expressed in terms of a differentiation with respect to μ . Consequently, (5) is not valid in this case.) If the IR cutoff represents some energy scale which decreases throughout the expansion of the universe, then from (5) one sees that $G_N(t)$ increases as a function of cosmic time. This raises a possibility of having the gravitational coupling G_N which is asymptotically free, as revealed in some quantum gravity models [21].

Setting $\mu = H$ in (5) and $\rho_m = \rho_{m0}a^{-3}$, and after combining (1) and (5) with the Friedmann equation for nonzero curvature, one obtains the following equation for the time evolution of the scale factor $a(t)$:

$$\dot{a}^2 - \gamma a \ddot{a} = \eta, \quad (6)$$

where γ and η are constants. The constant η is proportional to $\Omega_{k0} \equiv -k/H_0^2$, while γ does not depend on Ω_{k0} . The signs of these constants depend on the exact value of κ . Eq. (6) is a nonlinear second-order differential equation. In general, the explicit analytic solutions are difficult to find. However, below we present an analytical proof that there are no physically acceptable solutions with the property that the acceleration \ddot{a} changes the sign during the evolution.

First, consider the case $\eta = 0$. In this case, the solution can be found analytically. Requiring $a(0) = 0$, the solution is

$$a(t) = \left(\frac{t}{t_0}\right)^\alpha, \quad (7)$$

where $\alpha \equiv \gamma/(\gamma - 1) > 0$. (For $\alpha \leq 0$, the solution does not correspond to an expanding universe.) For $0 < \alpha < 1$, $\ddot{a} < 0$ everywhere except at $t \rightarrow \infty$. For $\alpha = 1$, $\ddot{a} = 0$ everywhere. For $1 < \alpha < 2$, $\ddot{a} > 0$ everywhere except at $t \rightarrow \infty$. For $\alpha = 2$, $\ddot{a} = 2/t_0^2$ is a constant. For $\alpha > 2$, $\ddot{a} > 0$ everywhere except at $t = 0$.

Now consider the case $\eta \neq 0$. If \ddot{a} changes the sign, then there exists a particular time t_1 at which $\ddot{a}(t_1) = 0$. Eq. (6) then implies $\dot{a}^2(t_1) = \eta$. This is clearly impossible for a negative η , so the rest of the discussion is restricted to a positive η . The expanding universe satisfying $\dot{a}^2(t_1) = \eta$ satisfies the initial condition $\dot{a}(t_1) = \sqrt{\eta}$, so one can verify that the solution of (6) reads

$$a(t) = a_1 + \sqrt{\eta}(t - t_1), \quad (8)$$

where a_1 is a free integration constant corresponding to an arbitrary choice of the initial condition $a(t_1) = a_1$. Clearly, the solution (8) has the property $\ddot{a}(t) = 0$ for each t . Thus,

if $\ddot{a}(t_1) = 0$ for *some* t_1 , then $\ddot{a}(t) = 0$ for *all* t . Consequently, \ddot{a} cannot change sign during the evolution, Q.E.D. [22].

Now we turn to discuss variable CC models in the light of the vacuum decay law as predicted by gravitational holography and given by (1). For a comprehensive list of proposals of various vacuum decay laws, see [23]. Cosmological implications of a few mostly discussed laws in the literature were critically examined in [24]. It was stressed in [24] that the renormalization-group (RG) running model of Shapiro *et al.* [25, 26] led to the most general cosmology for vacuum decaying universes. The running of ρ_Λ in [25] is of the form

$$\rho_\Lambda(\mu) = c_2\mu^2 + c_0, \quad (9)$$

where in [25] the RG scale was set at $\mu = H$, $c_2 \sim M_{Pl}^2$ and c_0 was the IR limit of the CC and represents here the true ground state of the vacuum [27]. The model phrased by (9) was based on the observation [28] that even a ‘true’ CC in conventional field theories could not be fixed to any definite constant (including zero) owing to the RG running effects. The variation of the CC arises solely from field fluctuations of the heaviest particles, without introducing any quintessence-like scalar fields. Particle contributions to the RG running of the CC which are due to vacuum fluctuations of massive fields have been properly derived in [29], with a somewhat peculiar outcome that more massive fields do play a dominant role in the running at any scale. The ‘coincidence problem’ is simply understood by noting that the scale associated to the CC is simply given as a geometrical mean of the largest and the smallest scale in the universe today. It was shown [26] that the law (9), in conjunction with the continuity equation (4), can explain recent cosmological observations. In the context of the present paper, however, we note a serious drawback of the model described by (9) and (4), when trying to accommodate the prediction from holography as given by (1). Namely, employing (4) implies that G_N is static, and insertion of the law (9) in (1) unavoidably sets the ground state of the vacuum to zero ($c_0 = 0$). This means that ρ_Λ from (9) scales as H^2 , leading to a constant r for spatially flat universes, as explained in detail in the first part of the paper. Thus, an attempt to bring the above model in accordance with holography, leads unavoidably to undesired phenomenological implications.

Now we try a different approach, in which the model given by (9) is investigated together with the continuity equation (3). In this case, the scale μ is not left to a free choice but is fixed by Eq. (5) [31]. From (1), (5) and (9) we find an explicit expression for the scale μ in

the form

$$\mu^2 = \frac{\sqrt{-c_0 \rho_m} - c_0}{c_2}. \quad (10)$$

The constants c_2 and c_0 can be expressed in terms of the present-day values of the cosmological parameters as

$$c_2 = \frac{\kappa}{G_0} \frac{1 + r_0}{r_0}, \quad (11)$$

$$c_0 = -\frac{\rho_{\Lambda 0}}{r_0}. \quad (12)$$

Using $\rho_m = \rho_{m0} a^{-3}$, we find from the above expressions that the ratio r now scales as (for arbitrary spatial curvature)

$$r = r_0 a^{-3/2}. \quad (13)$$

The ratio r becomes larger in the past, so we can expect a transition from acceleration to deceleration at late times.

By exploiting the Friedmann equation for flat space, one is allowed to express the scale μ in the alternative form as

$$\begin{aligned} \mu &= \left(\frac{3}{8\pi\kappa} \right)^{1/4} \left(\frac{-c_0}{c_2} \right)^{1/4} H^{1/2} \\ &= \sqrt{\frac{3}{8\pi\kappa(r_0 + 1)}} \sqrt{H_0 H}. \end{aligned} \quad (14)$$

Hence, if the model described by (9) and (3) respects the holographic prediction, the scale μ is fixed at $\mu \sim H^{1/2}$.

Solving the Friedmann equation for the scale factor $a(t)$, we obtain the solution as

$$a(t) = r_0^{2/3} \left(e^{\frac{3\beta}{2}t} - 1 \right)^{2/3}, \quad (15)$$

where $\beta \equiv H_0/(1+r_0)$. Note that $c_0 < 0$ [Eq.(12)] does not imply the anti-de Sitter universe, since G_N is also varying in our model. On the contrary, (15) shows that the universe in our model is asymptotically de Sitter, with $a(t) \simeq r_0^{2/3} e^{\beta t}$.

Now we are in a position to compare our scale (14), obtained in a consistent treatment, with the Hubble parameter and the inverse future horizon computed in the same model. For the most part of the evolution history of the universe the scales are plotted in Fig. 1. We find it indicative to have our scale (14) much closer to the inverse future horizon than to the Hubble parameter.

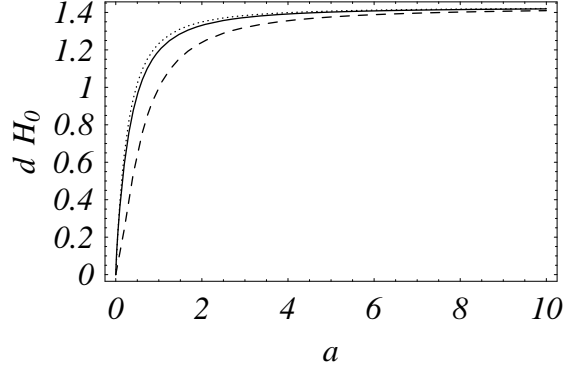


FIG. 1: The evolution of various cosmological scales d in units of H_0^{-1} as functions of a , all obtained from the model described by Eqs. (9), (1) and (5) for $r_0 = 3/7$. The future event horizon is represented by the dotted line, the scale $\mu^{-1}\sqrt{3/8\pi\kappa}$ given by (14) by the solid line and the Hubble distance by the dashed line.

One can also find that the gravitational coupling G_N scales in the above model as

$$G_N = G_{N0} \frac{r_0 + a^{3/2}}{r_0 + 1}, \quad (16)$$

so that the minimal value for $a = 0$ is $G_N = 0.3G_{N0}$ for $r_0 = 3/7$. We also find that the ratio \dot{G}_N/G_N is static, and is given by the value

$$\frac{\dot{G}_N}{G_N} = H_0 \frac{3}{2(1+r_0)}. \quad (17)$$

Eqs. (16) and (17) are at least marginally consistent with the bound on variation of the gravitational coupling from primordial nucleosynthesis [30], as well as with the upper bound from Solar Systems experiments for the quantity (17) [32].

Let us now study the effective dark-energy EOS. Since the matter expansion rate is modified by a scaling of G_N , we need to adopt the framework of the effective EOS (for ρ_Λ) put forward by Linder and Jenkins [33], in order to compare our model with observations. By extracting the standard matter contribution in the Friedmann equation, the rest is by definition the effective dark-energy density ρ_Λ^{eff} ,

$$H^2 = \frac{8\pi G_{N0}}{3} (\rho_{m0} a^{-3} + \rho_\Lambda^{eff}). \quad (18)$$

The effective EOS can then be defined as

$$w_{eff}(a) = -1 - \frac{1}{3} \frac{a}{\rho_\Lambda^{eff}} \frac{d\rho_\Lambda^{eff}}{da}. \quad (19)$$

One finds that the effective EOS for the present model is

$$w_{eff}(a) = -1 - \frac{r_0(1 - a^{3/2})}{(r_0 + a^{3/2})^2 - r_0(1 + r_0)} . \quad (20)$$

In particular, the present value corresponding to $a = 1$ is $w_{eff} = -1$. In fact, the qualitative behavior of w_{eff} is similar to that obtained for the running CC models which obey the equation of continuity (4) [24]. It is easy to convince one that $w_{eff} < -1$ for small z ($z = a^{-1} - 1$), but note that arguments leading to the Big Rip no longer apply to running CC models [24, 34, 35]. The reason for having $w_{eff} < -1$ here lies in the modified expansion rate for matter caused by the variable G_N , and not in the exotic nature of ρ_Λ .

Let us now briefly comment on a qualitative difference between the results presented here and those obtained in another class of generalized holographic dark energy models introduced recently by Elizalde *et al.* [35]. While we promote the Newton constant to a scale-dependent quantity, they identify the IR cutoff with some combination of natural IR cutoffs: the Hubble distance, the particle horizon distance, the future event horizon, or even the length scale associated with the cosmological constant or with the span of life of the universe (when the lifetime of the universe is finite). In the models mostly considered within the above class of holographic dark energy, a Big Rip sort of singularity is encountered within the effective phantom phase. It was shown though [35, 36, 37, 38] that quantum and quantum gravity effects might prevent evolution towards the Big Rip. In contrast, there is obviously no occurrence of the Big Rip singularity in our solution (15), and by closer inspection of the effective EOS (20) one sees that the effective phantom phase is only transient, approaching -1 from above in the limit $t \rightarrow \infty$. An intriguing possibility to unify the early-time inflation and late-time acceleration of the universe within the same model of the type [35] has emerged recently [39]. Now the bottom line is to have transitions between phantom and nonphantom phases with at least two phantom phases corresponding to inflation at early times and acceleration at late times. The nonphantom phase can then be identified with, e.g., matter-dominated era. In our model, however, a phantom-nonphantom transition at early time does not occur, and hence, without introducing additional degrees of freedom, our model is not capable to unify early-time inflation with late-time acceleration of the universe.

Finally, let us consider the transition redshift z_* , which denotes a switch of the universe from deceleration to acceleration, or equivalently, the redshift at which $\ddot{a} = 0$ in (15). One

finds $z_* = (r_0/2)^{-2/3} - 1$. Recent observations indicate $z_* < 0.72$ at 2σ [7]. The presence of baryons and possibly massive neutrinos in the matter sector of our model enables one to make r_0 as large as 0.35/0.65, but z_* less than about 1.4 still cannot be obtained. This is definitely the weakest point of our model. The problem here lies in the fact that the only free parameter of the model is r_0 (which reflects the unknown c_0), which is by observational data allowed to vary only by a very small amount. Thus in a given setup we have no available free parameters in the model to improve the above fit. Note also that all physical quantities do not depend on the parameter κ .

To this end, we make some suggestions how to modify the minimal setup explored above. Obviously, the minimal setup consistent with the generalized holographic energy density is not completely satisfactory when confronting the most recent observational data. One way is to add matter in the equation of continuity. Generally, we obtain

$$\dot{G}_N(\rho_\Lambda + \rho_m) + G_N\dot{\rho}_\Lambda + G_N(\dot{\rho}_m + 3H\rho_m) = 0. \quad (21)$$

Note that even for the simplest case $\rho_\Lambda = \text{const.}$, the scale μ is not fixed in (21). So, the scale and an additional parameter describing deviation of the matter density from its canonical form would be introduced as new parameters in the model. It would be interesting to see if compatibility between holography and observations can be achieved for some ‘standard’ choice for μ : the inverse Hubble distance, the inverse particle horizon or the inverse future horizon. Another suggestion would be to introduce a light scalar field with its potential in the model. These would change the equation of continuity, which now also requires the EOS of the scalar field. We note that having two different components of the vacuum energy in the dark-energy setup might be of some interest, see [10] and [35]. Still, in a running CC scenario accommodating holography such a construction is not appealing; a light degree of freedom is redundant there from the beginning.

To summarize, we have considered a phenomenological scale-dependence for the cosmological constant as predicted by gravitational holography. The role of the scaling parameter is played by the IR cutoff in such a manner that the information from quantum gravity be consistently encoded in ordinary quantum field theory. In the scenario in which also Newton’s constant scales with the same parameter, we have shown that the choice for the IR cutoff in the form of the Hubble parameter is not phenomenologically viable, either for spatially flat or for curved universes. The same scenario has been shown to have a potential

to univocally fix the scaling parameter when various variable CC models are constrained to be concordant with holography. We have shown that in this scenario the phenomenological viability of these models is substantially increased, although the minimal scenario is not quite satisfactory. We have indicated that models which might have a potential to improve the minimal scenario have much less predictive power.

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